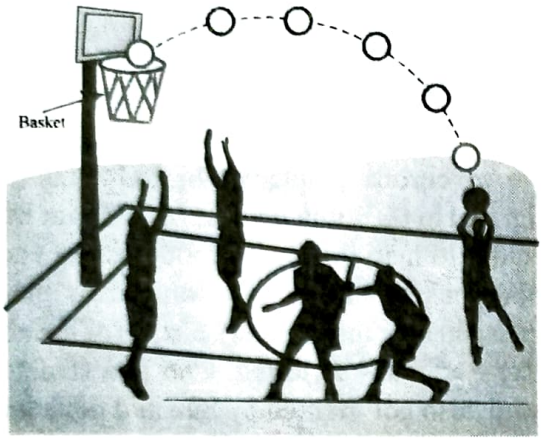
CASE-BASED QUESTIONS

1. Basketball is a very popular game and enthusiastic team sport. Two teams of five players each play this game on a rectangular court, usually indoors. Each team tries to score by tossing the ball through the opponents goal, an elevated horizontal hoop and net called a basket. One of player throws the ball for scoring, along a path shown in figure.



Read the above passage and answer the following questions.

(*a*) Name the type of polynomial whose geometrical representation is parabola.

(*i*) linear polynomial

(*ii*) cubic polynomial.

(*iii*) quadratic polynomial

(*iv*) none of these

(*b*) If the path of basketball is represented by x^2-2x-3, then its zeroes are

(*i*) $-1, 3$ (*ii*) $-1, 2$

(*iii*) $-2, 3$ (*iv*) $-3, 2$

(*c*) The axis of symmetry runs through the vertex. If the parabola open upwards, then the vertex of the quadratic polynomial has value

(*i*) maximum (*ii*) minimum

(*iii*) fixed (*iv*) none of these.

(*d*) Which of these is a polynomial for the path of basketball.

(*i*) $2x^3-6x^2+12x+8$ (*ii*) $3x+5$

(*iii*) x^2+5x+6 (*iv*) $x^2+5\sqrt{x}+10$

(*e*) The representation of the path of basketball as a polynomial with sum of zeroes as 0 and product of zeroes as -9 is

(*i*) x^2-x+9 (*ii*) x^2-9

(*iii*) x^2+9 (*iv*) x^2+x+9

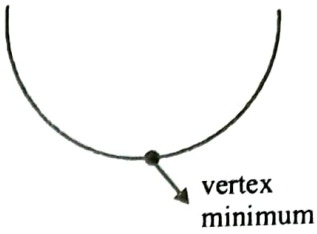
Sol. (*a*) Clearly (*iii*) holds

(*b*) $x^2-2x-3 = x^2-3x+x-3$

$= x(x-3)+1(x-3)$

$= (x-3)(x+1)$

$\therefore$ zeroes are $3, -1$ $\therefore$ (*i*) holds



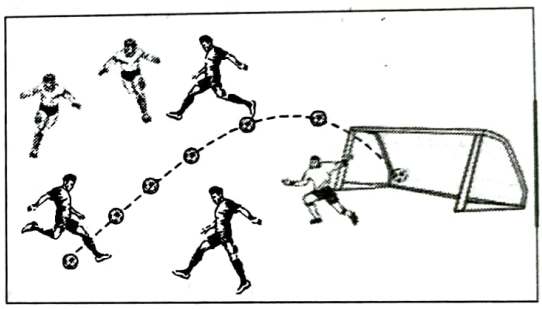
(*c*) clearly (*ii*) holds

(*d*) clearly (*iii*) holds

(*e*) clearly (*ii*) holds

[$\because$ sum of zeroes = 0 and product = -9)

2. In a Football match between France and Argentina, A player kicks the ball towards the goal along the path whose equation is $x^2 = 3x + 10$ as shown in the figure.



Read the above passage and answer the following questions

(*a*) Name the shape of the curve formed by the path

(*i*) Straight line (*ii*) Parabola

(*iii*) Circle (*iv*) Spiril

(*b*) Maximum number or zeroes of the given polynomial are

(*i*) 2 (*ii*) 3

(*iii*) 1 (*iv*) 0

(*c*) Product of zeroes of the polynomial $p(x) = x^2-3x-10$ is

(*i*) -10 (*ii*) 30

(*iii*) -3 (*iv*) 10

(*d*) Sum of zeroes of the polynomial is

(*a*) 4 (*ii*) 3

(*iii*) -3 (*iv*) 10

(*e*) The point of intersection of $p(x) = x^2 - 3x = 10$ with x-axis are

(*i*) $5, 0$ (*ii*) $-2, 0$

(*iii*) $5, -2$ (*iv*) $3, -1$

Sol. (*a*) clearly (*ii*) holds

(*b*) clearly (*i*) holds

(*c*) clearly (*i*) holds

(*d*) clearly (*iii*) holds

(e) clearly (*iii*) holds

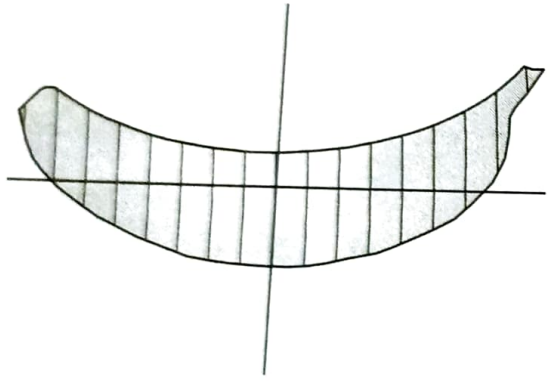
$[\because x^2-3x-10 = x^2-5x+2x-10$

$= x(x-5)+2(x-5) = (x-5)(x+2))$

3. The quadratic function can model the natural shape of a banana. Now we know that a parabola shape must have a quadratic function, therefore in standard form of $f(x) = ax^2 + bx + c$. To find an equation for the parabola shape of banana, we need to find the value of a, b and c. From the banana picture below we can see that a quadratic function is able to model the banana quite accurately with

$a = 0.1, b = 0, c = 0 \therefore$ the equation is $f(x) = 0.1x^2$.

Quadratic Function On a Banana



(*a*) Name the shape of banana curve from the given figure

(*i*) straight line (*ii*) parabola

(*iii*) circle (*iv*) none of these

(*b*) Find the number of zeroes of the polynomial for the shape of banana

(*i*) 1 (*ii*) 3

(*iii*) 2 (*iv*) 0

(*c*) If the curve of banana is represented by $f(x) = x^2 - x - 12$, find its zeroes

(*i*) 4, –3 (*ii*) –4, 3

(*iii*) 3, –1 (*iv*) –3, 1

(*d*) If the representation of banana curve where one zero is 4 and the sum of zeroes is 0, then find the quadratic

(*i*) x^2-2 (*ii*) x^2-4

(*iii*) x^2-16 (*iv*) x^2-8

(e) If the representation of banana curve whose sum of zeroes is 3 and product of zeroes is 2, then polynomial is

(*i*) x^2-3x-2 (*ii*) x^2-3x+2

(*iii*) x^2+3x-2 (*iv*) x^2+3x+2

Sol. (*a*) clearly (*ii*) holds

(*b*) clearly (*i*) holds

(*c*) $x^2-x-12 = x^2-4x+3x-12$

$= x(x-4)+3(x-4)$

$= (x-4)(x+3)$

$\therefore$ zeroes are 4, –3 $\therefore$ (*i*) holds

(*d*) Since one zero is 4 and sum of zeroes is '0', then reqd. polynomial is $x^2 - 16$ *i.e.* (*iii*) holds

(*e*) Since S = 3, P = 2

$\therefore$ reqd. polynomial is

$x^2 - Sx + P$ *i.e.* $x^2 - 3x + 2$. $\therefore$ (*ii*) holds

4. Due to corona pandemic in India many workers migrated to their village. Mr. M.S. Tyagi from Jaipur decides to help them with food packets and clothes. Food packets and clothes donated by them can be represented by the zeroes (*i.e.* α, β) of the polynomial $p(x) = x^2 - x - 2$. Rajan, who is a student of M.S. Tyagi, also got impired by him and donated the food packets and clothes in the form of a polynomial whose zeroes are $1 + 2\alpha$ and $1 + 2\beta$.

(*a*) Value of $p(x) = x^2 - x - 2$ at $x = -3$ is

(*i*) 11 (*ii*) –14

(*iii*) 4 (*iv*) 10

(*b*) Zeroes of the polynomial are

(*i*) 2, 1 (*ii*) 2, –1

(*iii*) –2, 1 (*iv*) –2, –1.

(*c*) Sum of zeroes of the polynomial whose zeroes are $1 + 2\alpha$ and $1 + 2\beta$ in the form of which Rajan donates food packets and clothes are

(*i*) 4 (*ii*) 0

(*iii*) –5 (*iv*) none of these

(*d*) Product of zeroes of the polynomial whose zeroes are $1 + 2\alpha$ and $1 + 2\beta$ is

(*i*) 4 (*ii*) 0

(*iii*) – 5 (*iv*) none of these

(*e*) Write the actual polynomial.

(*i*) $k(x^2 - 4x - 5)$ (*ii*) $k(x^2 + 4x - 5)$

(*iii*) $k(x^2 - 4x + 5)$ (*iv*) $k(x^2 + 4x + 5)$

Sol. (*a*) $p(x) = x^2 - x - 2$

$\therefore$ $p(-3) = 9 + 3 - 2 = 10$

$\therefore$ (*iv*) holds

(*b*) Since $x^2 - x - 2 = x^2 - 2x + x - 2$

$= x(x-2) + 1(x-2)$

$= (x-2)(x+1)$

$\therefore$ zeroes are 2, –1 $\therefore$ (*ii*) holds

(c) α, β and zeroes of $x^2 - x - 2$

$\therefore \alpha + \beta = 1, \alpha\beta = -2$

Now sum of zeroes *i.e.* S for new zeroes *i.e.* $1 + 2\alpha, 1 + 2\beta$

$= 1 + 2\alpha + 1 + 2\beta$

$= 2 + 2(\alpha + \beta) = 2 + 2(1) = 4$

$\therefore$ (*i*) holds

(d) Products of zeroes *i.e.* P

$= (1 + 2\alpha)(1 + 2\beta)$

$= 1 + 2(\alpha + \beta) + 4\alpha\beta$

$= 1 + 2(1) + 4(-2)$

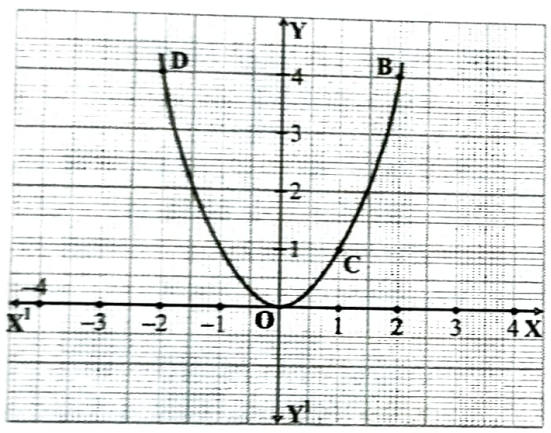
$= 3 - 8 = -5$ $\therefore$ (*iii*) holds

(e) Since $5 = 4, P = -5$

$\therefore$ new polynomial is

$k(x^2 - Sx + P) = k(x^2 - 4x - 5)$ $\therefore$ (*i*) holds

5. A bee flies after every 3 seconds Mr. Suraj recorded the position of the bee by the graph paper. At 11.00am. the location of bee was recorded at the point B. After 3 seconds the bee has moved to the position C.



(a) What is the distance between points B and C ?

(*i*) $\sqrt{3}$ cm (*ii*) $\sqrt{10}$ cm

(*iii*) 2

(*iv*) cannot be determined.

(b) After 3 more seconds the bee moves from C to a point on *x*-axis such that it covers the same distance *i.e.* BC. What can be the possible. Co-ordinates of the location of that point

(*i*) (4, 0), (–2, 0) (*ii*) (3, 0), (–1, 0)

(*iii*) (2, 0), (–2, 0) (*iv*) none of these.

(c) At some point of time, the bee sits on the point D (–2, 4). The point which divides C and D in the ratio. 1 : 2 will be

(*i*) on the *x*-axis (*ii*) In Ist quadrant

(*iii*) on the *y*-axis (*iv*) in IInd qudarant

(d) The area of the triangle formed by joining the points B, D and origin is

(*i*) 24 sq. units (*ii*) 16 sq.units

(*iii*) 4 sq.units (*iv*) 8 sq.units

(e) The point on the *y*-axis, which is equidistant from B and C is

(*i*) (2, 4) (*ii*) (2, 2)

(*iii*) (0, 3) (*iv*) (0, 2)

Sol. (a) Clearly co-ordinates of point B are (2, 4) and that of C are (1, 1)

$\therefore BC = \sqrt{(2-1)^2 + (4-1)^2} = \sqrt{1+9} = \sqrt{10}$ cm

$\therefore$ (*ii*) holds

(b) Let a point on the *x*-axis be $(x, 0)$

$= (x-1)^2 + (0-1)^2 = BC^2 = 10$

$\Rightarrow (x-1)^2 = 10 - 1 = 9$

$\Rightarrow x - 1 = \pm 3 = x = 1 \pm 3 = -2, 4$

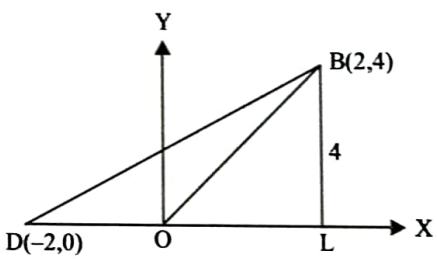
$\therefore$ reqd. points can be (–2, 0) or (4, 0) $\therefore$ (*i*) holds

(c) The point which divides CD in the ratio 1 : 2 is

$\left(\frac{-2+2}{1+2}, \frac{4+2}{1+2}\right)$ *i.e.* (0, 2), which lies on y – axis

$\therefore$ (*iii*) holds

(d) Area of Δ OBD



$= \frac{1}{2}(OD)(BL)$

$= \frac{1}{2}(2)(4) = 4$ sq. units

(e) Any point on *y*-axis is P $(0, y)$

$\therefore PB = PC \Rightarrow PB^2 = PC^2$

$(2-0)^2 + (4-y)^2 = (0-1)^2 + (1-y)^2$

$\Rightarrow 4 + 16 + y^2 - 8y = 1 + 1 + y^2 - 2y$

$\Rightarrow 6y = 18 \Rightarrow y = 3$

$\therefore$ P is (0, 3) $\therefore$ (*iii*) holds

6. In Urban Estate Phase-2, Mr. Neeraj opened a courier company. Charges for couriering a parcal are as follows

(*i*) Rs. 12 per Kg. and

(*ii*) a fixed pick-up service charges of Rs. 15. Seema sends a courier of w kg.

(a) Which of the following equation below show the correct relationship between courier charges 'C' and the weight 'w'.

(i) $C + 15 = 12 \times w$ (ii) $C = 15 + 12 \times w$

(iii) $C \times 15 = 12 \times w$ (iv) $C = 12 \times 6 \times w$

(b) If Seema paid Rs. 75, the weight of the courier is

(i) 5 kg (ii) 6 kg

(iii) 6-9 kg (iv) 8 kg.

(c) Raman, Kapil and Vijay are close friends. On Vijay birthday , Raman sent him a carrom board and Kapil sent him a cricket bat. The weight of the cricket bat was 4kg. How much money Kapil paid ?

(i) ₹ 57 (ii) ₹ 60

(iii) ₹ 63 (iv) ₹ 58

(d) On Vijay birthday, Raman sent him a carrom board with weight 3kg. How much money Raman paid ?

(i) ₹ 51 (ii) ₹ 60

(iii) ₹ 63 (iv) ₹58

(e) The weight of cricket bat was 1kg. more than weight of caram board.

How much more money was paid by Kapil ?

(i) ₹ 10 (ii) ₹ 8

(iii) ₹ 12 (iv) ₹15.

Sol. (a) Fixed charges = ₹15

Charges for weight 'w' = 12 × w.

∴ total charges 'C' are given by

$C = 15 + 12 \times w$.

∴ (ii) holds

(b) Since $C = 75$

$\therefore 75 = 15 + 12 \times w$

$\Rightarrow 60 = 12w \Rightarrow w = 5$ kg

∴ (i) holds

(c) Since $w = 4$ kg.

$\therefore C = 15 + 12 \times 4 = 15 + 48 = ₹63$

∴ (iii) holds

(d) Since $w = 4$ kg

$\therefore C = 15 + 12 \times 3 = ₹51$

∴ (i) holds

(e) Since extra weight = 1 kg.

∴ Extra money paid = 12 × 1 = ₹12.

∴ (iii) holds

7. In Chandigarh , Raju opened a Jai mata Taxi-service company. Charges for a Taxi are as follows

(i) Rs. 10 per km and

(ii) a fixed pick-up service charges Rs. 20.

Narinder fixed Taxi from Jai-Mata Taxi-service company. Suppose he travelled for x km.

(a) Which of the following equation below show the correct relationship between Taxi-service charges 'C' and the km. 'x'.

(i) $C = 20 + 10x$ (ii) $C + 20 = 10x$

(iii) $C = 10x + 10$ (iv) $C = 10 + 20x$

(b) If Narinder paid Rs. 120, the distance travelled by him is

(i) 15 km (ii) 10 km

(iii) 20 km (iv) 8 km

(c) Ram and Bhim both travelled by Jai-Mata Taxi-service. Ram travelled 50 kg and Bhim travelled 70 km. How much more money was paid by Bhim ?

(i) ₹ 150 (ii) ₹ 100

(iii) ₹ 200 (iv) ₹ 250

(d) Manoj and Kulbir travelled 80 km and 100 km respectivels. Total money paid to company was

(i) ₹ 1800 (ii) ₹ 1840

(iii) ₹ 1820 (iv) ₹ 1850.

(e) In festival season, company offered 10% discount on charges for km. covered. Manoj and Kulbir travelled 200 km and 150 km respectively.

Total money paid to company was

(i) ₹320 (ii) ₹350

(iii) ₹ 370 (iv) ₹ 400.

Sol. (a) $C = 20 + 10x$ *i.e.* (i) holds

(b) Since C = Rs 120

$\therefore 120 = 20 + 10x$

$\Rightarrow 10x = 100 \Rightarrow x = 10$. km. ∴ (ii) holds

(c) Extra money paid by Bhim

$= (70 - 50)\ 10 =$ Rs. 200

∴ (iii) holds

(d) Total amount paid

$= 20 + 80 \times 10 + 20 + 100 \times 10$

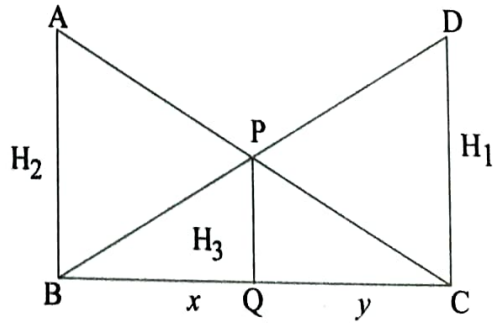
$= 820 + 1020 =$ Rs 1840

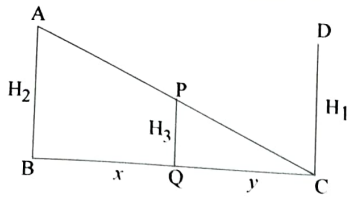
(e) Total Amount paid

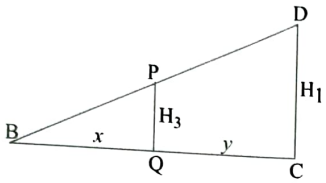
$$= 20 + \frac{200 \times 10 \times 10}{100} + \frac{150 \times 10 \times 10}{100}$$

$= 20 + 200 + 150 =$ Rs. 370

8. Three farmers are living happily in a village. One day there was a dispute between them and they separated their land by erecting pole at three boundary of their land. One day due to cyclone two poles broke down as shown in the fig. After watching the fig. they started living happily again and decided to measure length of the middle pole without measurement.







Read the above passage and answer the following questions

(*a*) (*i*) $H_3 = \frac{H_1 + H_2}{2}$ (*ii*) $H_3 = \frac{H_1 H_2}{H_1 + H_2}$

(*iii*) $H_3 = \frac{H_2 - H_1}{2}$ (*iv*) All of these.

(*b*) If AB = 7, DC = 4, then H_3 =(*i*) $\frac{28}{13}$

(*ii*) $\frac{28}{11}$

(*iii*) $\frac{26}{11}$ (*iv*) $\frac{23}{11}$.

(*c*) If AB = 7, PQ = 4, then H_1 =

(*i*) $\frac{19}{3}$ (*ii*) $\frac{23}{3}$

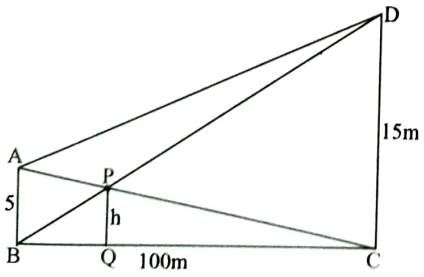
(*iii*) $\frac{26}{3}$ (*iv*) $\frac{28}{3}$

(*d*) If PQ = 12 m, DC = 16 m, then AB =

(*i*) 40 m. (*ii*) 42 m

(*iii*) 48 m. (*iv*) 52 m.

(*e*) If two poles 5 m and 15 m high are 100 m apart, then the height of the point of intersection of the line joining the top of each poles to the foot of the opposite side is



(*i*) 2.75 m (*ii*) 3.75 m

(*iii*) 1.75 m (*iv*) 4.75 m.

Sol. (*a*) clearly Δ ABC and ΔPQC are similar (by A.A. rule)

$$\therefore \frac{H_2}{H_3} = \frac{x+y}{y} \text{ i.e. } \frac{H_3}{H_2} = \frac{y}{x+y}$$

Clearly Δ BCD and Δ BQP are similar

$$\frac{H_1}{H_3} = \frac{x+y}{x} \text{ i.e. } \frac{H_3}{H_1} = \frac{x}{x+y}$$

$$\therefore \frac{H_3}{H_2} + \frac{H_3}{H_1} = \frac{y+x}{x+y} = 1$$

$$\Rightarrow H_3\left(\frac{1}{H_2} + \frac{1}{H_1}\right) = 1 \Rightarrow H_3 = \frac{H_1 H_2}{H_1 + H_2}$$

$\therefore$ (*ii*) holds

(*b*) $H_3 = \frac{H_1 H_2}{H_1 + H_2}$. Here $H_2 = 7$, $H_1 = 4$

$$= \frac{7 \times 4}{7+4} = \frac{28}{11} \therefore (iv) \text{ holds}$$

(*c*) $H_3 = \frac{H_1 H_2}{H_1 + H_2} \Rightarrow H_3 H_1 + H_3 H_2 = H_1 H_2$

$$\Rightarrow H_1 (H_2 - H_3) = H_2 H_3$$

$$\Rightarrow H_1 = \frac{H_2 H_3}{H_2 - H_3} = \frac{28}{3} \begin{bmatrix} H_2 = 7 \\ H_3 = 4 \end{bmatrix}$$

$\therefore$ (*iv*) holds

(*d*) Since $H_3 = \frac{H_1 H_2}{H_1 + H_2}$.

Here $H_3 = 12$; $H_1 = 16$

$AB = H_2 = ?$

$\Rightarrow H_1 H_3 + H_3 H_2 = H_1 H_2$

$\Rightarrow H_2 (H_1 - H_3) = H_1 H_3$

$$\Rightarrow H_2 = \frac{H_1 H_3}{H_1 - H_3} = \frac{16 \times 12}{16 - 12} = 48 \text{m.}$$

$\therefore$ (*iii*) holds

(*e*) $h = \frac{5 \times 15}{5+15} = \frac{75}{20} = \frac{15}{4} = 3.75$ m

$\therefore$ (*ii*) holds

9. A computer animation below shows a cat moving in a straight line. Its height h meters abut the ground is given by $8t - 3h = -9$, where t is the time in seconds after it starts moving. In the same animation, a mouse starts to move at the same time as the cat and its movement is given has $-3t + h = 1$.